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Oscillations in thin nematic layers

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In the present paper a thin nematic liquid crystal layer between two identical boundary surfaces (solid walls or free surfaces in the case of a freely suspended film) is considered. In a mean field approximation it is shown that the interference between the boundary surface-induced smectic density waves results in oscillations of the free energy of the nematic layer and disjoining pressure acting on the boundary surfaces. Theoretical dependence of disjoining pressure on the nematic layer thickness is in qualitative agreement with experiment. Also we have considered a thin film of polar nematic in which in addition to an ordinary monolayer smectic A phase (S_{A_1}) with the layer thickness d equal to the molecular length l the partial bilayer smectic A phase (S_{A_d}) occurs. It is shown that the variation of the distance between the boundary surfaces can result in the oscillatory S_{A_1} - A_{A_d} phase transitions in this nematic film

1. Introduction

It is known that the surface has an effect on the structure of liquids and liquid crystals and that this effect propagates for some distance into the liquid. This results in the appearance of surface layers with properties different from those of the bulk phase of liquid. For example, the liquid-solid substance interface not only enhances the orientational ordering in nematics and imposes some orientational order on the isotropic phase of liquids having a nematic phase [1-6] but also induces orientational ordering in non-mesogeneous liquids [7]. Furthermore, both the nematic free surface and the nematic-solid substrate interface have an effect on the positional molecular order in liquids that leads to the appearance of a layered smectic A structure [1, 8-13].

The thin layer of liquid or nematic between two identical boundary surfaces (solid walls or free surfaces in the case of a freely suspended film) is particularly interesting. When two such surfaces are sufficiently close together then the interference between liquid surface layers takes place which results in the appearance of extra phenomena. These phenomena have been clearly revealed by Horn *et al.* [1] from measurements of the forces between two molecularly smooth surfaces of mica separated by the nematogen 4-*n*-pentyl-4'-cyanobiphenyl (5CB) in both the planar and homeotropic orientations. It has been found that there is a short-range force which oscillates as a function of the nematic film thickness between attraction and repulsion and the period of oscillation is equal to the molecular length l and molecular diameter a at the homeotropic and planar orientations, respectively. Also Horn *et al.* [1] have established that the amplitude of oscillations decays exponentially with the distance between the two mica surfaces. It should be added that similar oscillations with a period equal to the molecular size have been observed in isotropic liquids composed of more or less spherical molecules [14, 15]. The qualitative explanation of such a phenomenon was also given by Horn *et al.* [1]. They attributed the oscillations of the

force between the two mica surfaces to the solid wall-induced smectic ordering of molecules in the liquid crystal layer. As for its theoretical description, up to now this has been based on either computer simulation methods [16] or the analysis of model systems quite different from liquids and liquid crystals [17].

We propose here a simple description of the oscillations in the thermodynamic properties of thin nematic layers between two identical boundary surfaces. In a mean field approximation it is shown that the interference between boundary surface-induced smectic density waves results in oscillations of the free energy of the liquid crystal layer and disjoining pressure acting on the boundary surfaces. Theoretical dependence of the disjoining pressure on the nematic film thickness is in qualitative agreement with experiment [1]. We also consider a thin film of a polar nematic in which in addition to the ordinary monolayer smectic A phase (S_{A_1}) with the layer thickness d equal to the molecular length l the partial bilayer smectic A phase (S_{A_d}) with $l < d < 2l$ occurs. It is shown that the variation of the distance between the boundary surfaces can result in oscillatory S_{A_1} - S_{A_d} phase transitions in this nematic film.

2. Oscillations in thin nematic layers as a result of interference between boundary surfaces-induced smectic density waves

We consider the nematic layer between two identical boundary surfaces (solid walls or free surfaces in the case of a freely suspended film). For simplicity we assume that the orientational order in the liquid crystal is ideal (i.e. the long axes of all molecules are oriented parallel to the director \mathbf{n}). Furthermore, the director \mathbf{n} is assumed to be aligned along the z axis which is normal to the boundary surfaces (i.e. the nematic layer has a homeotropic orientation). The layer thickness is equal to $2L$ ($z=0$ and $z=2L$ at the first and second boundary surfaces, respectively).

When the layer under consideration is infinite ($2L \rightarrow \infty$) and homogeneous and we do not take into account the interaction between the mesogenic molecules and the boundary surfaces then its properties in a mean field approximation are completely described by the single particle distribution function [18]

$$\rho(z) = 1 + 2\sigma_1 \cos(2\pi z/l), \quad (1)$$

where σ_1 is the coordinate independent monolayer smectic order parameter (for the present only a classic monolayer smectic A phase (S_{A_1}) is assumed to be possible in the sample). The free energy density in such a layer is given by the Landau expansion:

$$f_0(\sigma_1) = f_N + A_1\sigma_1^2 + C_1\sigma_1^4 + \dots, \quad (2)$$

where f_N is the free energy density in the nematic phase. $A_1 = \alpha_1(T - T_1)$, $\alpha_1 = \text{constant}$, T_1 is the S_{A_1} -N transition temperature in an infinite and homogeneous liquid crystal, C_1 is constant. When $T > T_1$, then $A_1 > 0$ and the nematic phase is observed ($\sigma_1 = 0$). When $T < T_1$, then $A_1 < 0$ and the smectic A phase is energetically more favourable than the nematic phase and $\sigma_1 \neq 0$. Since we consider the nematic layer then $T_1 < T$ and $A_1 > 0$.

Now let us assume that the layer is sufficiently thick ($2L \rightarrow \infty$) and take into account the interaction between the mesogenic molecules and the boundary surfaces. In this case we can neglect the mutual influence of the boundary surfaces and consider separately each half of the layer adjacent to one of the boundary surfaces. For simplicity we suppose that the molecules interact with the boundary surfaces due to short range

attractive forces and the corresponding interaction potentials for first ($z=0$) and second ($z=2L$) boundary surfaces, respectively can be represented as

$$G_1(z) = -G_0\delta(z), \quad (3)$$

$$G_2(z) = -G_0\delta(z-2L), \quad (4)$$

where G_0 is a constant and $\delta(z)$ is the Dirac function. If we assume that each boundary surface induces its own smectic A structure (i.e. density wave) which decays completely with propagation into the bulk of the layer then the corresponding single particle distribution functions for the first and second halves of the layer are given, respectively, by

$$\rho_1(z) = 1 + 2\sigma_1^{(1)}(z) \cos(2\pi z/l), \quad (5)$$

$$\rho_2(z) = 1 + 2\sigma_1^{(2)}(z) \cos(2\pi(z-2L)/l), \quad (6)$$

where both $\sigma_1^{(1)}(z)$ and $\sigma_1^{(2)}(z)$ are the corresponding z coordinate dependent density wave amplitudes (the sample is assumed to be homogeneous in the xy plane and the xy dependence of both $\sigma_1^{(1)}$ and $\sigma_1^{(2)}$ can be neglected). Then the functional (2) for the density of the free energy in the layer can be rewritten as [19]

$$f(\sigma_1(z)) = f_N + A_1\sigma_1^2(z) + C_1\sigma_1^4(z) + K_1\left(\frac{d\sigma_1}{dz}\right)^2 + \dots, \quad (7)$$

where K_1 is analogous to an elastic constant. If the temperature of the layer is much higher than the S_{A_1} -N transition temperature in an infinite and homogeneous sample then the term $\sim \sigma_1^4(z)$ in equation (7) can be neglected. In this case the minimization of the functional (7) with respect to $\sigma_1(z)$ leads to the simple differential equation

$$\frac{d^2\sigma_1(z)}{dz^2} - \frac{1}{\xi_1^2}\sigma_1(z) = 0 \quad (8)$$

where $\xi_1 = (K_1/A_1)^{1/2}$ is the longitudinal correlation length of the S_{A_1} phase fluctuations in the nematic phase. The solutions of equation (8) for the first and second halves of the layer are

$$\sigma_1^{(1)}(z) = \sigma_{10} \exp(-z/\xi_1), \quad (9)$$

$$\sigma_1^{(2)}(z) = \sigma_{10} \exp((z-2L)/\xi_1). \quad (10)$$

For a thick ($L/\xi_1 \rightarrow \infty$) nematic layer the interaction energy between the mesogenic molecules and the boundary surfaces per unit area is

$$(F_{\text{surf}})_{S_{A_1}} = \int_0^{2L} n[\rho_1(z)G_1(z) + \rho_2(z)G_2(z)]dz = -2nG_0 - 4nG_0\sigma_{10} \quad (11)$$

and the total free energy per unit area is

$$F_{S_{A_1}} = \int_0^{2L} f(\sigma_1(z))dz + (F_{\text{surf}})_{S_{A_1}} = F_N + 2\xi_1 A_1 \sigma_{10}^2 - 4nG_0\sigma_{10}, \quad (12)$$

where F_N is the total free energy per unit area in the nematic phase and n is the average number of molecules per unit volume. The minimization of the total free energy with respect to σ_{10} leads to the following expressions for σ_{10} and $F_{S_{A_1}}$

$$\sigma_{10} = g/2, \quad (13)$$

$$F_{S_{A_1}} = F_N - \frac{1}{2}\xi_1 A_1 g^2, \quad (14)$$

where

$$g = 2nG_0/\xi_1 A_1.$$

From this equation we can conclude that the existence of the smectic A structures at the boundary surfaces of a thick layer is always energetically favourable and this result is independent of the layer thickness.

Finally, let us consider a thin nematic layer between the boundary surfaces when the magnitude of the ratio L/ξ_1 is finite and we cannot neglect the mutual influence of the boundary surfaces (i.e. we cannot consider separately each half of the layer adjacent to one of the boundary surfaces). In this case the solution of equation (8) must be the superposition of both solutions (9) and (10) and the smectic density wave in the layer must be the superposition of two boundary surface-induced density waves. Therefore, we take the expression

$$\sigma_1(z) = \sigma_{10} [\exp(-z/\xi_1) + \exp((z-2L)/\xi_1)] \quad (15)$$

satisfying the condition of a minimum for $\sigma_1(z)$ at the middle of the layer

$$\left. \frac{d\sigma_1(z)}{dz} \right|_{z=L} = 0 \quad (16)$$

as the solution to equation (8) and represent the single particle distribution function as

$$\begin{aligned} \rho(z) = & 1 + 2\sigma_{10} \exp(-z/\xi_1) \cos(2\pi z/l - \psi_1) \\ & + 2\sigma_{10} \exp((z-2L)/\xi_1) \cos(2\pi(z-2L)/l - \psi_2) \end{aligned} \quad (17)$$

where both ψ_1 and ψ_2 are constant phase shifts introduced to take into account the mutual influence of two boundary surface-induced density waves. It should be noted that the normalization of the one particle distribution function (17) is not necessary because of the space between the two boundary surfaces is assumed to be open (molecules can come into it and leave it). Now the energy of interaction between the mesogenic molecules and the boundary surfaces per unit area is given by

$$\begin{aligned} (F_{\text{surf}})_{S_{A_1}} = & -2\sigma_{10}nG_0 [\cos \psi_1 + \exp(-2L/\xi_1) \cos(4\pi L/l - \psi_1) \\ & + \cos \psi_2 + \exp(-2L/\xi_1) \cos(4\pi L/l + \psi_2)]. \end{aligned} \quad (18)$$

Minimization of this expression with respect to both ψ_1 and ψ_2 leads to

$$\psi_1 = -\psi_2 = \text{tn}^{-1} \left[\frac{\exp(-2L/\xi_1) \sin(4\pi L/l)}{1 + \exp(-2L/\xi_1) \cos(4\pi L/l)} \right]. \quad (19)$$

Then the total free energy per unit square is

$$\begin{aligned} F_{S_{A_1}} = & \int_0^{2L} f(\sigma_1(z)) dz + (F_{\text{surf.}})_{S_{A_1}} = F_N + 2\xi_1 A_1 \\ & \times \{ \sigma_{10}^2 (1 - \exp(-4L/\xi_1)) - \sigma_{10} g [\cos \psi_1 + \exp(-2L/\xi_1) \cos(4\pi L/l - \psi_1)] \}. \end{aligned} \quad (20)$$

After minimization of this expression with respect to σ_{10} we obtain

$$\sigma_{10} = \frac{1}{2} g [\cos \psi_1 + \exp(-2L/\xi_1) \cos(4\pi L/l - \psi_1)] [1 - \exp(-4L/\xi_1)]^{-1}, \quad (21)$$

$$\begin{aligned} (F_{S_{A_1}} - F_N)/(\xi_1 A_1) = & -\frac{1}{2} g^2 [\cos \psi_1 + \exp(-2L/\xi_1) \cos(4\pi L/l - \psi_1)]^2 \\ & \times [1 - \exp(-4L/\xi_1)]^{-1} \end{aligned} \quad (22)$$

It is seen from these equations that both the surface smectic order parameter σ_{10} and the free energy of the layer are oscillatory functions of the distance $2L$ between the two boundary surfaces. The qualitative explanation of the results obtained can be as follows. Both the smectic order parameter and the free energy of the layer are determined by the interaction between the mesogenic molecules and the boundary surfaces. When the interference between the boundary surface-induced smectic density waves is constructive at these surfaces then such an interaction is enhanced. When this interference is destructive, then the interaction between the molecules and the boundary surfaces grows weak. Since the result of the interference between two coherent waves oscillates with the variation of the distance between their sources (the boundary surfaces) then both the smectic order parameter and the free energy of the layer must be oscillatory functions of the layer thickness $2L$.

One of the most important characteristics of the system under consideration is the disjoining pressure acting on the boundary surfaces. The extra disjoining pressure due to the boundary surface-induced smectic A structure in the nematic film is determined by

$$\Delta P = - \left(\frac{\delta(F_{S_{A_1}} - F_N)}{\delta 2L} \right)_T. \quad (23)$$

Since $F_{S_{A_1}} - F_N$ is an oscillatory function of the layer thickness $2L$ then the extra disjoining pressure must oscillate with the variation of the layer thickness. The results of numerical calculations for the extra disjoining pressure via the equations (19), (22) and (23) (the longitudinal correlation length ξ_1 is assumed to be equal to the molecular length l) is shown in figure 1. It is seen that the period of the disjoining pressure oscillations is equal to the period of the smectic density wave l and the amplitude of the oscillations decays exponentially with the nematic film thickness $2L$. The characteristic

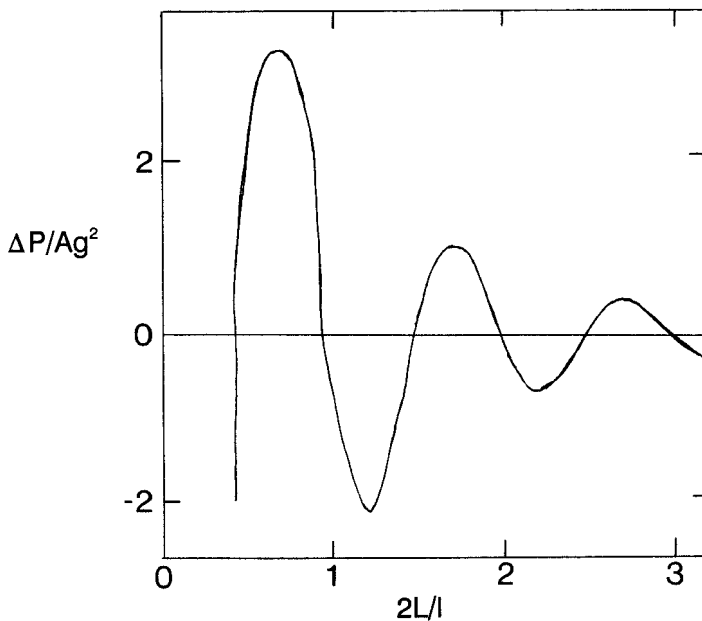


Figure 1. The dependence of the disjoining pressure on the distance between the boundary surfaces. $\xi_1 = l$.

decay length is equal to the longitudinal correlation length of the S_{A_1} phase fluctuations in the nematic phase. These theoretical results are in agreement with experiment. In addition the positions of the maxima and minima of the theoretical curve in figure 1 coincide qualitatively with experiment [1]. It should be noted that as we approach the S_{A_1} -N transition temperature T_1 in an infinite and homogeneous sample the longitudinal correlation length $\xi_1 \rightarrow \infty$ and the oscillations of the disjoining pressure can be observed at sufficiently large distances between the boundary surfaces. This result is also in agreement with experiment [1] in which the oscillations of the disjoining pressure were observed in the smectic A phase of 4-*n*-octyl-4'-cyanobiphenyl (8CB) at a distance between the boundary surfaces equal to about $2\ \mu\text{m}$ which corresponds to the thickness of more than 600 smectic layers.

3. Oscillatory S_{A_1} - S_{A_d} phase transitions in thin polar nematic layers

In addition to the ordinary monolayer smectic A phase (S_{A_1}) the partial bilayer smectic A phases (S_{A_d}) with a layer thickness $l < d < 2l$ exist in some strongly polar liquid crystals consisting of molecules having end groups (CN or NO_2) with large permanent dipole moments ($\approx 4\ \text{D}$) [20–22]. In such liquid crystals the monolayer and partial bilayer smectic A phases are either separated by the reentrant nematic phase or the direct A_{A_1} - S_{A_d} phase transition is observed [20, 22].

A microscopic model of the S_{A_d} phase has been proposed by de Jeu and Longa [23]. According to this model the S_{A_d} phase is composed not only of individual polar molecules (monomers) but also their pairs (dimers) with a zero total electric dipole moment. However de Jeu and Longa's theory is in contradiction with measurements of the dielectric anisotropy of polar liquid crystals [24]. Dielectric studies of some polar systems [25] show the existence of long range antiferroelectric order in the S_{A_d} phase. Therefore, it should be reasonable to extend de Jeu and Longa's model for the bilayer smectic A phase (S_{A_2}) with long range antiferroelectric order [26] to the S_{A_d} phase. Then the S_{A_d} phase can be represented as the smectic A structure shown in figure 2. It is seen that each layer of this structure consists of two macroscopic sublayers with mutually opposite alignments of the molecular dipoles and the smectic layer thickness d satisfies the condition $l < d < 2l$. In a mean field approximation such smectic A structure in polar liquid crystals with perfect orientational order can be described by the single particle distribution function

$$\rho(z, s) = 1 + 2\sigma_2 s \cos(2\pi z/d), \quad (24)$$

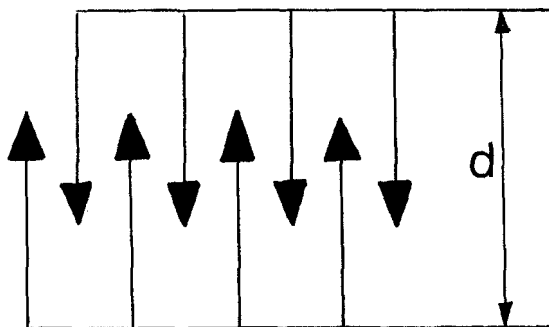


Figure 2. The partial bilayer smectic A phase (A_{A_d}) with long range antiferroelectric order in a polar liquid crystal. ($l < d < 2l$).

where σ_2 is the partial bilayer smectic A order parameter and s is the variable defining the molecular orientation ($s = +1$ for the polar molecules with the dipole moment aligned parallel to the director (up) and $s = -1$ for the polar molecules aligned in the opposite direction (down)). A Landau expansion for the free energy density in the S_{Ad} phase can be obtained directly from equation (7) if the following replacements are made:

$$\sigma_1(z) \rightarrow \sigma_2(z), \quad A_1 \rightarrow A_2 = \alpha_2(T - T_2), \quad C_1 \rightarrow C_2, \quad K_1 \rightarrow K_2$$

where T_2 is the S_{Ad} -N transition temperature in an infinite and homogeneous liquid crystal sample.

Now we consider the interaction between the polar liquid crystal molecules and the boundary surfaces. We must take into account the fact that some interfaces such as the free surface not only impose a homeotropic orientation but also induce polar ordering of these molecules. For example, in the first surface layer the polar molecular heads are aligned into the liquid crystal bulk and the aliphatic tails towards the free surface. The next layer has the opposite orientation and so on. Thus the interface can induce not only the ordinary monolayer S_{A_1} phase but also the smectic A structure with long range antiferroelectric order, such as the S_{Ad} phase. A possibility of the polar ordering was first discussed by Parsons [27]. In this paper the interaction energy between the polar mesogenic molecules and surface due to the polar ordering is supposed to be $\sim -(\mathbf{n} \cdot \mathbf{v})$ where \mathbf{v} is the unit vector normal to the boundary surface. When perfect orientational order takes place in polar liquid crystals and we take into account the boundary surface-induced polar ordering then the interaction potentials between the molecules and first and second boundary surfaces, respectively, can be written as

$$G_1(z, s) = -G_0\delta(z) - sG_1\delta(z), \quad (25)$$

$$G_2(z, s) = -G_0\delta(z - 2L) + sG_1\delta(z - 2L). \quad (26)$$

Here we suppose that the polar ordering is due to short range forces.

Following the development in the previous section first we consider a sufficiently thick polar nematic layer ($2l \rightarrow \infty$) between two independent boundary surfaces. Each boundary surface induces its own S_{Ad} phase which decays completely with the propagation into the bulk of the liquid crystal sample. Then the corresponding single particle distribution functions for this S_{Ad} phase can be represented as

$$\rho_1(z, s) = 1 + 2\sigma_{20} \exp(-z/\xi_2) s \cos(2\pi z/d), \quad (27)$$

$$\rho_2(z, s) = 1 - 2\sigma_{20} \exp((z - 2L)/\xi_2) s \cos(2\pi(z - 2L)/d), \quad (28)$$

where $\xi_2 = (K_2/A_2)^{1/2}$ is the longitudinal correlation length for the S_{Ad} phase fluctuations in the nematic phase. Here the exponential decay of the partial bilayer smectic order parameters results from the Euler-Lagrange equation which is completely equivalent to equation (8) when the temperature of the layer is much higher than the S_{Ad} -N transition temperature in an infinite and homogeneous sample.

Finally, if we proceed to consider a polar nematic layer of finite thickness (L/ξ_2 is the finite value) when we cannot neglect the mutual influence of the boundary surfaces then by analogy with the previous section the expression

$$\sigma_2(z) = \sigma_{20} [\exp(-z/\xi_2) + \exp((z - 2L)/\xi_2)] \quad (29)$$

should be taken as the solution of the Euler–Lagrange equation for $\sigma_2(z)$ and a single particle distribution function describing the S_{A_d} phase can be represented as

$$\rho(z, s) = 1 + 2\sigma_{20} \exp(-z/\xi_2) s \cos(2\pi z/d - \psi_1^1) - 2\sigma_{20} \exp((z-2L)/\xi_2) s \cos(2\pi(z-2L)/d - \psi_2^1). \quad (30)$$

Then the contribution of the interaction between polar mesogenic molecules and the boundary surfaces to the total free energy is given by

$$(F_{\text{surf.}})_{S_{A_d}} = \frac{1}{2}n \sum_{s=\pm 1} \int_0^{2L} dz \rho(z, s) [G_1(z, s) + G_2(z, s)] = -2nG_0 - 2\sigma_{20}nG_1 [\cos \psi_1^1 - \exp(-2L/\xi_2) \cos(4\pi L/d - \psi_1^1) + \cos \psi_2^1 - \exp(-2L/\xi_2) \cos(4\pi L/d + \psi_2^1)]. \quad (31)$$

Minimization of this expression with respect to both ψ_1^1 and ψ_2^1 leads to

$$\psi_1^1 = -\psi_2^1 = -\text{tn}^{-1} \left[\frac{\exp(-2L/\xi_2) \sin(4\pi L/d)}{1 - \exp(-2L/\xi_2) \cos(4\pi L/d)} \right]. \quad (32)$$

Then the total free energy of the polar layer is determined by

$$F_{S_{A_d}} = \int_0^{2L} f(\sigma_2(z)) dz + (F_{\text{surf.}})_{S_{A_d}} = F_N + 2\xi_2 A_2 \{ \sigma_{20}^2 [1 - \exp(-4L/\xi_2)] - \sigma_{20} g^1 [\cos \psi_1^1 - \exp(-2L/\xi_2) \cos(4\pi L/d - \psi_1^1)] \}, \quad (33)$$

where

$$g^1 = 2nG_1/\xi_2 A_2.$$

After the minimization of this expression with respect to σ_{20} we obtain

$$\sigma_{20} = \frac{1}{2}g^1 [\cos \psi_1^1 - \exp(-2L/\xi_2) \cos(4\pi L/d - \psi_1^1)] [1 - \exp(-4L/\xi_2)]^{-1}, \quad (34)$$

$$(F_{S_{A_d}} - F_N)/(\xi_2 A_2) = -\frac{1}{2}g^2 [\cos \psi_1^1 - \exp(-2L/\xi_2) \cos(4\pi L/d - \psi_1^1)]^2 \times [1 - \exp(-4L/\xi_2)]^{-1}. \quad (35)$$

Thus the free energy of the boundary surface-induced S_{A_d} phase in the polar liquid crystal layer is also an oscillatory function of the distance $2L$ between the two boundary surfaces.

If in the polar liquid crystal both the monolayer (S_{A_1}) and partial bilayer (S_{A_d}) smectic A phases can exist then the boundary surfaces can induce both these smectic A phases in a thin nematic film. Since the free energies of these smectic A phases are oscillatory functions of the film thickness $2L$ (see expressions (22) and (35)) then the S_{A_1} phase is energetically more favourable at certain values of this thickness whereas other values of $2L$ correspond to the occurrence of the S_{A_d} phase. Thus varying the distance between two boundary surfaces we can obtain an alternation of the S_{A_1} and S_{A_d} phases, i.e. we observe oscillatory S_{A_1} – S_{A_d} phase transitions. The points of these phase transitions determined from the condition $F_{S_{A_1}} = F_{S_{A_d}}$ can be detected via the sharp discontinuities of the disjoining pressure acting on the boundary surfaces. This statement is illustrated in figure 3 in which the corresponding dependence of the disjoining pressure on the distance between the boundary surfaces calculated from equations (22), (23) and (35) is shown (for simplicity we set $A_1 = A_2$, $\xi_1 = \xi_2 = 1$, $g = g^1$ and $d = 1.51$). It is seen that in this case the disjoining pressure is not a simple oscillatory

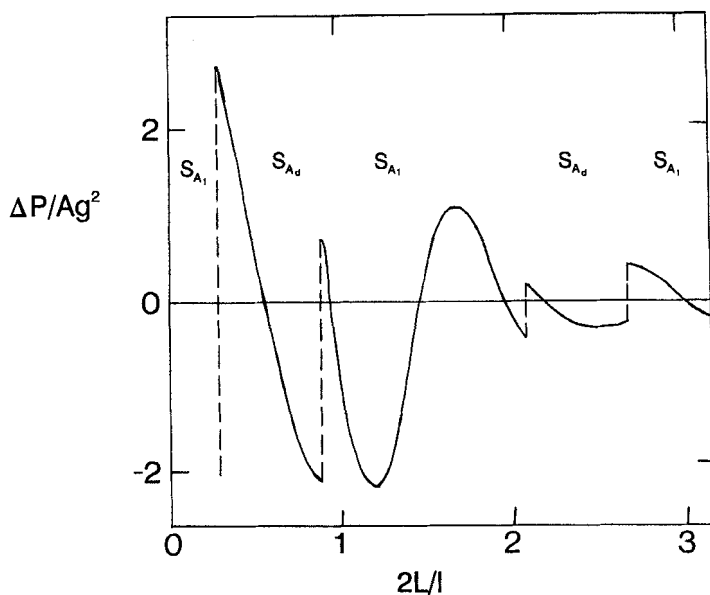


Figure 3. The layer thickness dependence of the disjoining pressure acting on the boundary surfaces of a polar nematic, $A_1 = A_2$, $\xi_1 = \xi_2 = l$, $g = g^1$, $d = 1.51$.

function with a single oscillation period. It consists of the pieces of two different oscillatory functions (with the oscillation periods equal to 1 and $d = 1.51$) and we can observe sharp discontinuities at the points of connections of these pieces (the S_{A_1} - S_{A_d} transition points). The magnitude of the discontinuities decays exponentially with the distance between the boundary surfaces. It should be added that oscillatory S_{A_1} - S_{A_d} phase transitions can also be detected via calorimetric and dielectric measurements.

Finally, we wish to note that the well-known polar mesogenic compound 4-n-octyloxybenzoyloxy-4'-cyanostilbene (T8) seems to be a good candidate for the experimental observation of such phenomena because the fluctuations of both the S_{A_1} and S_{A_d} phases coexist simultaneously in its reentrant nematic phase [28].

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